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LISP 2 Storage Management:

The "Growing Pain" Problem

ABSTRACT

This document presents an illustration of the "growing pain" problem, which relates to storage management functions in LISP 2. Four general approaches to a solution of the problem are presented.

1. INTRODUCTION

This document presents an illustration of the "growing pain" problem, which relates to storage management functions in LISP 2. Four general approaches to a solution of the problem are presented.

The problem of reallocating storage space in LISP 2 is analogous to the problem presented here--that of deriving an algorithm for rearranging a set of blocks in some pre-defined sequence. In the analogy given below, the yellow blocks represent free space to be allocated; brown blocks represent filled spaces (i.e., spaces that cannot be altered); and the "special" block represents space whose size is to be expanded.

2. DESCRIPTION OF THE PROBLEM

Imagine a collection of blocks arranged in a row. The blocks are all the same size. Some of the blocks are glued together. When two blocks are glued together, they may not be separated under any circumstances, and the order of the blocks may not be changed. Imagine that some of the blocks are yellow and the rest are brown, and that a yellow block is never glued to any other block. Thus, only brown blocks may be glued together. Note that contiguous brown blocks are not necessarily glued together. Imagine that one of the brown blocks is special. The special block must not be glued on both sides, though it may be glued on one. We assume that there is at least one yellow block in the collection, and there is only one special block.

An example of such a collection appears in Figure 1. Blocks that are not glued together are separated by a space. An arrow appears below the special block in the figure. There is one yellow block in the figure; this is indicated by a "Ø" in the appropriate box.

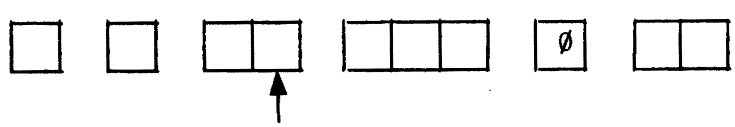

Figure 1. Growing Pain Problem Illustration

The following game of solitaire may be played with such a collection of blocks. The blocks are arranged in a row in some order. A player confronted with the arrangement of the blocks must rearrange the blocks so that a yellow block touches the special brown block. To simplify rearranging the blocks, the glued blocks may be unglued. Any rearrangement of the original situation may be achieved by interchanging a brown block and a yellow block. After the blocks are rearranged, the blocks that were unglued are rejoined. Two blocks that were attached originally must be attached together in the original order so that the separating of the blocks during the interchanging is invisible in the final rearrangement. After the blocks have been rearranged, a score is computed by counting up the number of interchanges of brown and yellow blocks necessary to form the rearrangement.

The computation of the score may be simplified as follows. Let N denote the number of brown blocks displaced one or more positions as a result of the rearrangement. If, in the final arrangement, a yellow block occupies a position originally occupied by a brown block, then the optimal score for the rearrangement is N. Otherwise the optimal score is N+1. Yellow blocks cannot be distinguished, so that rearrangements that have yellow blocks in corresponding positions are identical rearrangements.

An example of a rearrangement of the blocks illustrated in Figure 1 appears below. This rearrangement may be achieved in 7 interchanges.

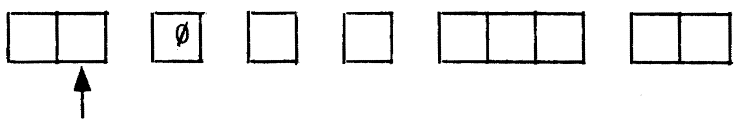

Figure 2. A Solution with Score 7

Though the game is fairly easy to play, the problem of finding a rearrangement of the blocks with a minimum score is a fairly involved combinatorial problem.

3. POSSIBLE APPROACHES

We wish to have a general algorithm that will enable us to play the game optimally for any collection of blocks. In the following paragraphs we will discuss a number of possible, though admittedly non-optimal approaches. Due to the symmetry of the game, it is sufficient to assume that the yellow block is to be moved to the right side of the special block. In the following discussions we will assume we are dealing with a right-side situation.

One general approach to the game is to pick one of the yellow blocks and move it toward the special block by interchanging the yellow block with an adjacent brown block. In other words, if a yellow block is somewhere to the right of the special block, and if all the blocks between the special block and the yellow block are moved one position to the right, then the yellow block will appear in its desired position, on the right of the special block. If a yellow block is somewhere to the left of the special block, then the game is solved by moving all the intervening blocks, including the special block, to the left. In our example, this approach yields a solution with a score of 3. In general, this approach will not yield an optimal solution.

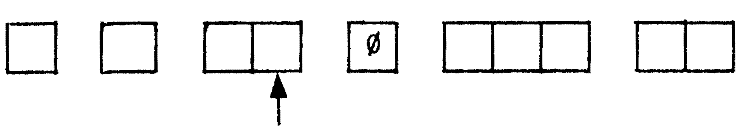

Figure 3. A Solution with Score 3

Another general approach to the game is to find two sets of blocks satisfying the following conditions. Each set of blocks must be a contiguous sequence of blocks and must contain all blocks glued to blocks in the set. The first set contains the special block and does not contain a yellow block. The second set contains at least one yellow block. Let N denote the number of blocks in the first set. If the number of blocks in the second set is N+1, then the game may be solved with a score of at most 2N+1. The player moves the brown blocks from each set to the other set by successively interchanging the yellow block into alternative sets. The goal of interchanging the blocks is to leave the yellow block in the second set with the special block to its left. The following diagrams illustrate this approach applied to our example.

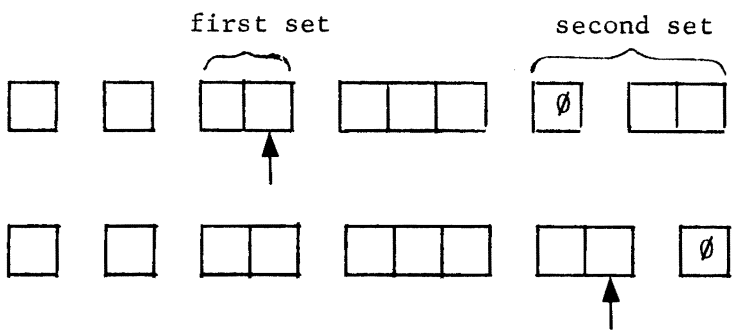

Figure 4. A Solution with Score 4

It should be noted that this approach can yield a solution at least as good as the previous approach, since the latter approach corresponds to the special case where the first set is contained in the second set. This approach may be optimized by considering all possible first and second sets. For each pair satisfying the specified constraints it is necessary to find a best score. The problem of finding a best score in this limited situation is simplified by the constraints on forming the sets.

A third approach is to find a set of blocks which contains all blocks glued to blocks in the set, together with the special block. The set should also have the property that a yellow block is adjacent to the rightmost of the blocks in the set. If there are N blocks in the set, then the game can be solved with a score of at most N+1 by permuting the blocks in the set so as to move the special block to the rightmost of the set. The following diagram illustrates one result of this approach when applied to our example.

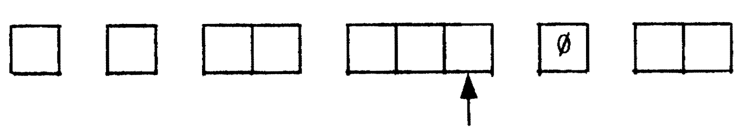

Figure 5. A Solution with Score 6

The third approach is a special case of the second approach, with an emphasis on rearranging the blocks in the limited situation in order to achieve an optimal score.

The approaches presented above may be characterized as efforts to find a direct solution after reducing the scope of the game. For example, after finding two sets satisfying the conditions, the game is reduced to a consideration of only the blocks in the two sets. All blocks not in the reduced game are thereby ignored in the computation of the score.

A fourth approach is to use heuristics and general problem-solving techniques such as goal reduction. One possible heuristic is to move a yellow block closer to the special block by interchanging it with a brown block not attached on either side. In the particular example, there is such a brown block to the left of the special block. This is interchanged with the yellow block, and then the first approach described above is applied, with the result that the special block is shifted to the left.

Figure 6 illustrates the original, the intermediate and the final arrangement of the blocks when a heuristic approach is applied to the sample game.

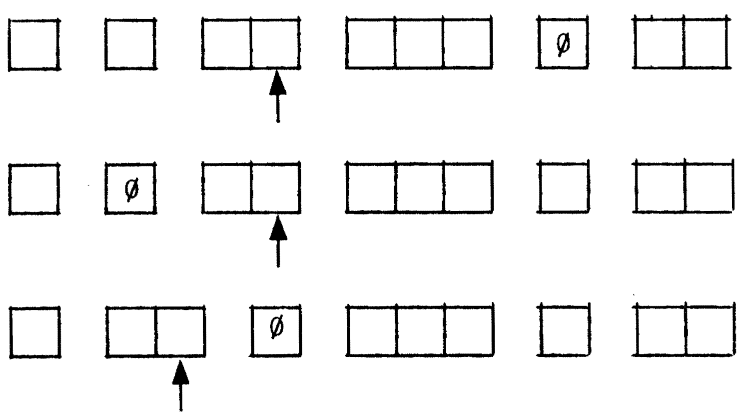

Figure 6. A Solution with Score 3

4. CONCLUSION

The four approaches described above indicate ways in which the "growing pain" problem could be handled in the design of the LISP 2 system. Further study of these and other approaches is necessary to determine which one will be the most fruitful.